Fundamental Bounds on Dissipation Factor for Wearable and Implantable Antennas

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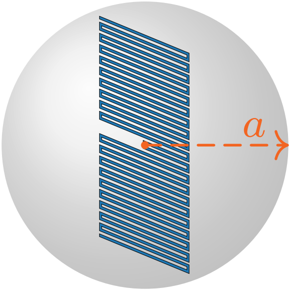
Outline



Electrically small antenna inside a circumscribing sphere of a radius a.

- Document available at `capek.elmag.org`.
- To see the graphics in motion, open this document in Adobe Reader!

Radiation Efficiency and Dissipation Factor

Radiation efficiency[1]:

$$\eta_{\mathrm{rad}} = \frac{P_{\mathrm{rad}}}{P_{\mathrm{rad}} + P_{\mathrm{lost}}} = \frac{1}{1 + \delta_{\mathrm{lost}}} \tag{1}$$

Dissipation factor[2] δ:

$$\delta_{\mathrm{lost}} = \frac{P_{\mathrm{lost}}}{P_{\mathrm{rad}}} \tag{2}$$

- fraction of quadratic forms (can be scaled with resistivity model).

[1] *145-2013 – IEEE Standard for Definitions of Terms for Antennas*, IEEE, 2014

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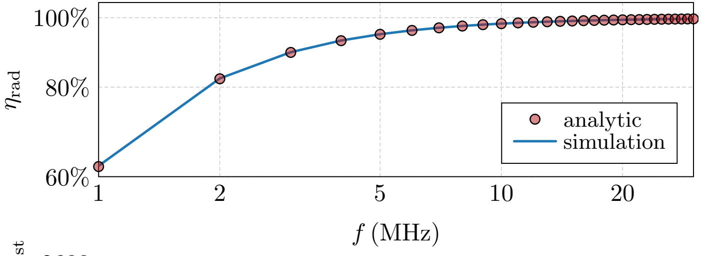
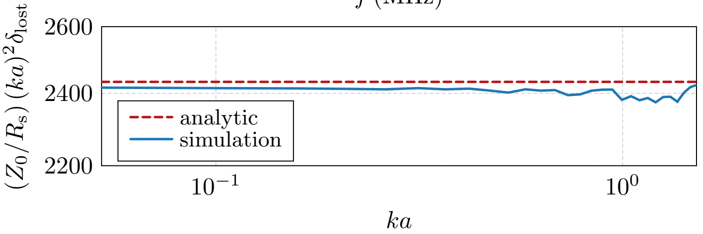
[1] *145-2013 – IEEE Standard for Definitions of Terms for Antennas*, IEEE, 2014

[2] R. F. Harrington, "Effect of antenna size on gain, bandwidth, and efficiency," *J. Res. Nat. Bur. Stand.*, vol. 64-D, pp. 1–12, 1960

Radiation Efficiency and Dissipation Factor: Example

A wire dipole of length $\ell = 5\,\mathrm{m}$ made of copper wire of $2.055\,\mathrm{mm}$:





What Is This Talk About?

Questions to be investigated...

1. What are the fundamental bounds on radiation effiency?
2. What are other costs (self-resonance, trade-offs)?
3. Are these bounds feasible?

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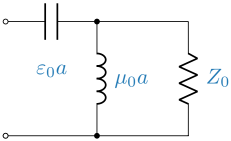
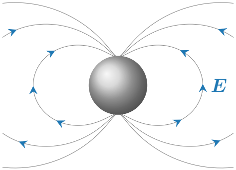
3. Are these bounds feasible?

Tools we have:

- Circuit quantities (equivalent circuits).
- Field quantities (spherical harmonics).
- Source currents (eigenvalue problems).



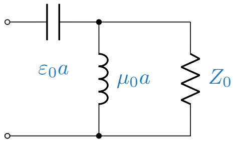


$$L_{mn} = \langle \boldsymbol{\psi}_m(\boldsymbol{r}), \mathcal{L}[\boldsymbol{\psi}_n(\boldsymbol{r})] \rangle$$
$$\mathbf{L} = [L_{mn}]$$
$$\mathbf{AI} = \lambda \mathbf{IB}$$

A Little History of the Problem. . .

Circuit Quantities

- Circuit quantities (equivalent circuits):
  1. C. Pfeiffer, “Fundamental efficiency limits for small metallic antennas,” *IEEE Trans. Antennas Propag.*, vol. 65, pp. 1642–1650, 2017.
  2. H. L. Thal, “Radiation efficiency limits for elementary antenna shapes,” *IEEE Trans. Antennas Propag.*, vol. 66, no. 5, pp. 2179–2187, 2018.



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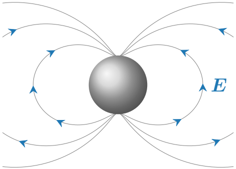
Field Quantities

- Field quantities (spherical harmonics):
  1. R. F. Harrington, "Effect of antenna size on gain, bandwidth, and efficiency," *J. Res. Nat. Bur. Stand.*, vol. 64-D, pp. 1–12, 1960.
  2. A. Arbabi and S. Safavi-Naeini, "Maximum gain of a lossy antenna," *IEEE Trans. Antennas Propag.*, vol. 60, pp. 2–7, 2012.
  3. K. Fujita and H. Shirai, "Theoretical limitation of the radiation efficiency for homogenous electrically small antennas," *IEICE T. Electron.*, vol. E98C, pp. 2–7, 2015.
  4. A. K. Skrivervik, M. Bosiljevac, and Z. Sipus, "Fundamental limits for implanted antennas: Maximum power density reaching free space," *IEEE Trans. Antennas Propag.*, vol. 67, no. 8, pp. 4978 –4988, 2019.



A Little History of the Problem. . .

Source Currents

- Source currents (eigenvalue problems):
    1. M. Uzsoky and L. Solymár, "Theory of super-directive linear arrays," *Acta Physica Academiae Scientiarum Hungaricae*, vol. 6, no. 2, pp. 185–205, 1956.
    2. R. F. Harrington, "Antenna excitation for maximum gain," *IEEE Trans. Antennas Propag.*, vol. 13, no. 6, pp. 896–903, 1965.
    3. M. Gustafsson, D. Tayli, C. Ehrenborg, *et al.*, "Antenna current optimization using MATLAB and CVX," *FERMAT*, vol. 15, no. 5, pp. 1–29, 2016.
    4. L. Jelinek and M. Capek, "Optimal currents on arbitrarily shaped surfaces," *IEEE Trans. Antennas Propag.*, vol. 65, no. 1, pp. 329–341, 2017.

$$L_{mn} = \langle \boldsymbol{\psi}_m(\boldsymbol{r}), \mathcal{L}[\boldsymbol{\psi}_n(\boldsymbol{r})] \rangle$$
$$\mathbf{L} = [L_{mn}]$$
$$\mathbf{A}\mathbf{I} = \lambda \mathbf{I}\mathbf{B}$$

Integral Operators and Their Algebraic Representation

Radiated and reactive power:

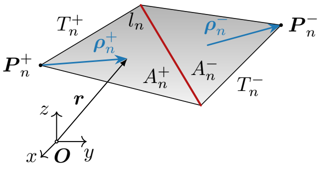
$$P_{\mathrm{rad}} + 2\mathrm{j}\omega\left(W_{\mathrm{m}} - W_{\mathrm{e}}\right) = \frac{1}{2}\left\langle \boldsymbol{J}\left(\boldsymbol{r}\right), \mathcal{Z}\left[\boldsymbol{J}\left(\boldsymbol{r}\right)\right]\right\rangle$$

Lost power (surface resistivity model):

$$P_{\mathrm{lost}} = \frac{1}{2}\left\langle \boldsymbol{J}\left(\boldsymbol{r}\right), \mathrm{Re}\left\{Z_{\mathrm{s}}\right\}\boldsymbol{J}\left(\boldsymbol{r}\right)\right\rangle$$

- The same approach as with the method of moments[3] (MoM)

$$\boldsymbol{J}\left(\boldsymbol{r}\right) \approx \sum_n I_n \boldsymbol{\psi}_n\left(\boldsymbol{r}\right)$$



RWG basis function $\boldsymbol{\psi}_n$.

[3] R. F. Harrington, *Field Computation by Moment Methods*. Piscataway, New Jersey, United States: Wiley – IEEE Press, 1993

Algebraic Representation of Integral Operators

Radiated and reactive power

$$P_{\mathrm{rad}} + 2\mathrm{j}\omega \left(W_{\mathrm{m}} - W_{\mathrm{e}}\right) = \frac{1}{2} \left\langle \boldsymbol{J}\left(\boldsymbol{r}\right), \mathcal{Z}\left[\boldsymbol{J}\left(\boldsymbol{r}\right)\right]\right\rangle \tag{3}$$

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Electric Field Integral Equation[4] (EFIE), $\mathbf{Z} = [Z_{mn}]$:

$$Z_{mn} = \int\limits_{\Omega} \boldsymbol{\psi}_m \cdot \mathcal{Z}\left(\boldsymbol{\psi}_n\right) \mathrm{d}S = \mathrm{j}kZ_0 \int\limits_{\Omega} \int\limits_{\Omega} \boldsymbol{\psi}_m\left(\boldsymbol{r}_1\right) \cdot \mathbf{G}\left(\boldsymbol{r}_1, \boldsymbol{r}_2\right) \cdot \boldsymbol{\psi}_n\left(\boldsymbol{r}_2\right) \mathrm{d}S_1 \, \mathrm{d}S_2. \tag{4}$$

[4] W. C. Chew, M. S. Tong, and B. Hu, *Integral Equation Methods for Electromagnetic and Elastic Waves.* Morgan & Claypool, 2009

Algebraic Representation of Integral Operators

Radiated and reactive power

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- Dense, symmetric matrix.
- An output from PEC 2D MoM code.

[4] W. C. Chew, M. S. Tong, and B. Hu, *Integral Equation Methods for Electromagnetic and Elastic Waves*. Morgan & Claypool, 2009

Algebraic Representation of Integral Operators

Lost power

$$P_{\mathrm{lost}} = \frac{1}{2} \langle \boldsymbol{J}(\boldsymbol{r}), \mathrm{Re}\{Z_{\mathrm{s}}\} [\boldsymbol{J}(\boldsymbol{r})] \rangle \tag{5}$$

Algebraic Representation of Integral Operators

Lost power

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$$L_{mn} = \int\limits_{\Omega} \boldsymbol{\psi}_m \cdot \boldsymbol{\psi}_n \, \mathrm{d}S \tag{6}$$

Surface resistivity model:

$$Z_{\mathrm{s}} = \frac{1 + \mathrm{j}}{\sigma \delta} \tag{7}$$

with skin depth $\delta = \sqrt{2/\omega \mu_0 \sigma}$.

Algebraic Representation of Integral Operators

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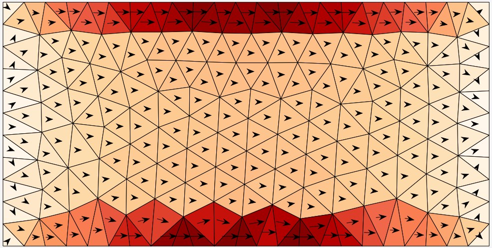
with skin depth $\delta = \sqrt{2/\omega \mu_0 \sigma}$.

- Sparse matrix (diagonal for non-overlapping functions $\{\boldsymbol{\psi}_m(\boldsymbol{r})\}$).
- The entries L_{mn} are known analytically.

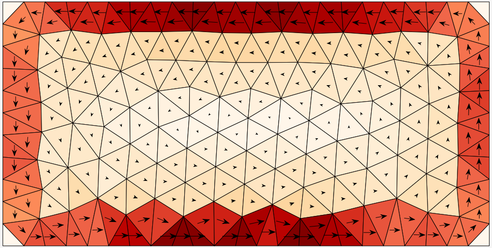
A Note: MoM Solution × Current Impressed in Vacuum

MoM solution



Solution to $\mathbf{I} = \mathbf{Z}^{-1}\mathbf{V}$ for an incident plane wave.

Current impressed in vacuum



Solution to $\mathbf{X}\mathbf{I}_i = \lambda_i \mathbf{R}\mathbf{I}_i$ (the first inductive mode).

A current can be chosen completely freely, only the excitation $\mathbf{V} = \mathbf{Z}\mathbf{I}$ may not be realizable.

Fundamental Bounds as QCQP Problems

- The optimization problems $\mathcal{P}_1$ and $\mathcal{P}_2$ can rigorously be formulated.

Maximum radiation efficiency

Problem $\mathcal{P}_1$:

$$\begin{aligned} \text{minimize} \quad & P_{\mathrm{loss}} \\ \text{subject to} \quad & P_{\mathrm{rad}} = 1 \end{aligned}$$

Maximum self-resonant radiation efficiency

Problem $\mathcal{P}_2$:

$$\begin{aligned} \text{minimize} \quad & P_{\mathrm{loss}} \\ \text{subject to} \quad & P_{\mathrm{rad}} = 1 \\ & \omega \left(W_{\mathrm{m}} - W_{\mathrm{e}} \right) = 0 \end{aligned}$$

Fundamental Bounds as QCQP Problems

- The optimization problems $\mathcal{P}_1$ and $\mathcal{P}_2$ can rigorously be formulated.
- Having quadratic forms for the physical quantities, the antenna metrics may be optimized.

Maximum radiation efficiency

Problem $\mathcal{P}_1$:

$$\begin{aligned} &\text{minimize} && \mathbf{I}^{\mathrm{H}}\mathbf{L}\mathbf{I} \\ &\text{subject to} && \mathbf{I}^{\mathrm{H}}\mathbf{R}\mathbf{I} = 1 \end{aligned}$$

Maximum self-resonant radiation efficiency

Problem $\mathcal{P}_2$:

$$\begin{aligned} &\text{minimize} && \mathbf{I}^{\mathrm{H}}\mathbf{L}\mathbf{I} \\ &\text{subject to} && \mathbf{I}^{\mathrm{H}}\mathbf{R}\mathbf{I} = 1 \\ & && \mathbf{I}^{\mathrm{H}}\mathbf{X}\mathbf{I} = 0 \end{aligned}$$

[5] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, Great Britain: Cambridge University Press, 2004

Fundamental Bounds as QCQP Problems

- The optimization problems $\mathcal{P}_1$ and $\mathcal{P}_2$ can rigorously be formulated.
- Having quadratic forms for the physical quantities, the antenna metrics may be optimized.
- The problems $\mathcal{P}_1$ and $\mathcal{P}_2$ are quadratically constrained quadratic programs[5] (QCQP).

Maximum radiation efficiency

Problem $\mathcal{P}_1$:

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Problem $\mathcal{P}_2$:

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Solution to Radiation Efficiency Bound ($\mathcal{P}_1$)

Lagrangian reads

$$\mathcal{L}(\lambda, \mathbf{I}) = \mathbf{I}^{\mathrm{H}} \mathbf{L} \mathbf{I} - \lambda \left(\mathbf{I}^{\mathrm{H}} \mathbf{R} \mathbf{I} - 1 \right). \tag{8}$$

Stationary points

$$\frac{\partial \mathcal{L}}{\partial \mathbf{I}^{\mathrm{H}}} = \mathbf{L}\mathbf{I} - \lambda \mathbf{R}\mathbf{I} = 0 \tag{9}$$

are solution to generalized eigenvalue problem (GEP):

$$\mathbf{L}\mathbf{I}_i = \lambda_i \mathbf{R}\mathbf{I}_i. \tag{10}$$

Substituting a discrete set of stationary points $\{\mathbf{I}_i, \lambda_i\}$ back to (8) and minimizing gives

$$\min_{\{\mathbf{I}_i\}} \mathcal{L}(\lambda, \mathbf{I}) = \lambda_1. \tag{11}$$

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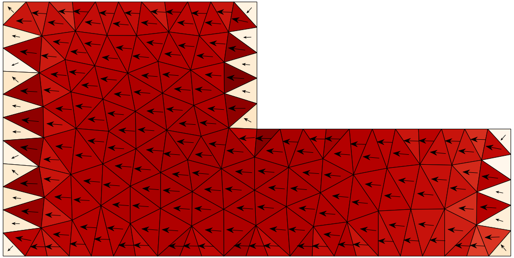
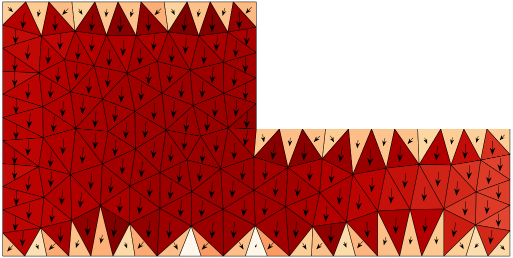
Example: Radiation Efficiency Bound of an L-plate ($\mathcal{P}_1$)

$ka = 1$, $R_\mathrm{s} = 0.01\,\Omega/\square$.



Optimal current (1st mode), $Z_0/R_\mathrm{s}\,(ka)^2\,\delta_\mathrm{loss} = 17.6$.



The 2nd current mode, $Z_0/R_\mathrm{s}\,(ka)^2\,\delta_\mathrm{loss} = 19.2$.

- Implicitly solved by dominant radiation mode[6] or simplification of EFIE[7].

[6] K. Schab, "Modal analysis of radiation and energy storage mechanisms on conducting scatterers," PhD thesis, University of Illinois at Urbana-Champaign, 2016

[7] M. Shahpari and D. V. Thiel, "Fundamental limitations for antenna radiation efficiency," *IEEE Trans. Antennas Propag.*, vol. 66, no. 8, pp. 3894–3901, 2018

Solution to Self-Resonant Radiation Efficiency Bound ($\mathcal{P}_2$)

The same solving procedure as with problem $\mathcal{P}_1$, two Lagrange multipliers, however:

$$\mathcal{L}\left(\lambda_1, \lambda_2, \mathbf{I}\right) = \mathbf{I}^{\mathrm{H}}\mathbf{L}\mathbf{I} - \lambda_1 \left(\mathbf{I}^{\mathrm{H}}\mathbf{R}\mathbf{I} - 1\right) - \lambda_2 \mathbf{I}^{\mathrm{H}}\mathbf{X}\mathbf{I}. \tag{12}$$

Stationary points

$$\left(\mathbf{L} - \lambda_2 \mathbf{X}\right) \mathbf{I}_i = \lambda_{1,i} \mathbf{R}\mathbf{I}_i. \tag{13}$$

Solving strategy:

1. Determine interval[8] of λ_2 such that $\mathbf{L} - \lambda_2 \mathbf{X} \succ \mathbf{0}$ (since $\mathbf{R} \succ \mathbf{0}$).
2. Solve (13) iteratively, pick the first minimum ($i = 1$) and maximize dual function $g = \sup \mathcal{L}\left(\lambda_{1,i}, \lambda_2, \mathbf{I}_i\right) = \max\limits_{\lambda_2} \lambda_{1,1}$.

Solution to Self-Resonant Radiation Efficiency Bound ($\mathcal{P}_2$)

The same solving procedure as with problem $\mathcal{P}_1$, two Lagrange multipliers, however:

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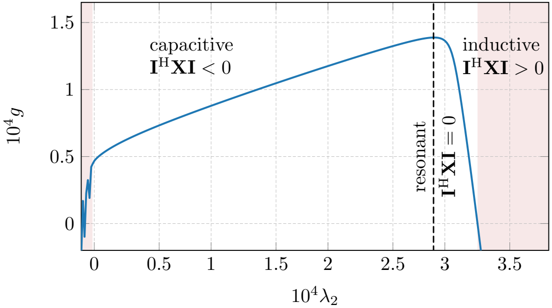
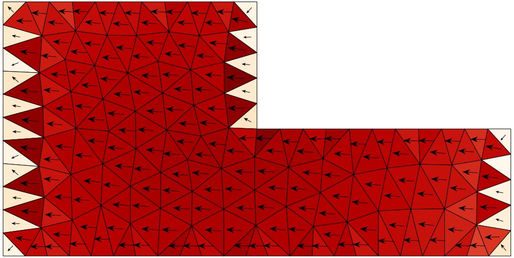


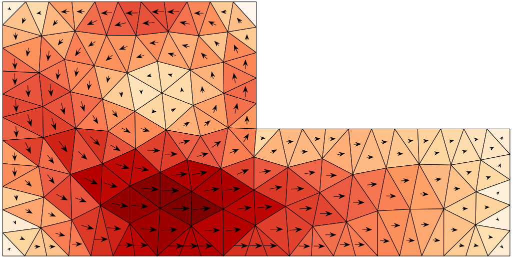
[8] M. Gustafsson and M. Capek, “Maximum gain, effective area, and directivity,” *IEEE Trans. Antennas Propag.*, vol. 67, no. 8, pp. 5282 –5293, 2019

Example: Optimal Currents for L-Shape Plate ($\mathcal{P}_1$ & $\mathcal{P}_2$)

$ka = 1$, $R_\mathrm{s} = 0.01\,\Omega/\square$.



Optimal current for $\mathcal{P}_1$,
$Z_0/R_\mathrm{s}\,(ka)^2\,\delta_\mathrm{loss} = 17.6$.



Optimal current for $\mathcal{P}_2$,
$Z_0/R_\mathrm{s}\,(ka)^4\,\delta_\mathrm{loss} = 52.3$.

Example: Optimal Currents for L-Shape Plate ($\mathcal{P}_1$ & $\mathcal{P}_2$)

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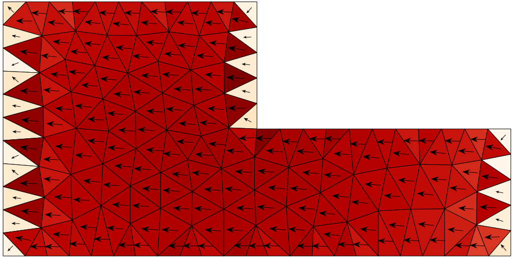

Optimal current for $\mathcal{P}_1$,
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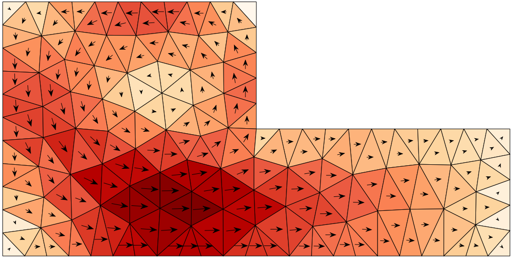

Optimal current for $\mathcal{P}_2$,
$Z_0/R_\mathrm{s}\,(ka)^4\,\delta_\mathrm{loss} = 52.3$.

The same optimization approach may be applied for any representation of the integral operators.

- Surface MoM, separable bodies, volumetric MoM, hybrid integral methods.

Exact Solution for a Spherical Shell ($\mathcal{P}_1$ & $\mathcal{P}_2$)

Spherical waves $\mathbf{u}_\alpha^{(1)}$, $\alpha = \{\tau, \sigma, m, l\}$ diagonalize all the operators, *i.e.*,

$$\left\langle \mathbf{u}_\alpha^{(1)}, \mathcal{Z}\left[\mathbf{u}_{\alpha'}^{(1)}\right] \right\rangle = p_\alpha \delta_{\alpha\alpha'} \tag{14}$$

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- Solution found by setting all waves to radiate unitary power, $\left\langle \mathbf{u}_\alpha^{(1)}, \mathcal{R}\left[\mathbf{u}_{\alpha'}^{(1)}\right] \right\rangle = 2\delta_{\alpha\alpha'}$.

Problem $\mathcal{P}_1$ ($ka \ll 1$)

- Dominant TM mode

$$\min_{\mathbf{I}} \delta_{\mathrm{loss}} = \frac{9}{4} \frac{R_\mathrm{s}}{Z_0} \frac{1}{(ka)^2}.$$

Problem $\mathcal{P}_2$ ($ka \ll 1$)

- TM and TE modes tuned to resonance

$$\min_{\mathbf{I}} \delta_{\mathrm{loss}} = 3 \frac{R_\mathrm{s}}{Z_0} \frac{1}{(ka)^4}.$$

Exact Solution for a Spherical Shell ($\mathcal{P}_1$ & $\mathcal{P}_2$)

Spherical waves $\mathbf{u}_\alpha^{(1)}$, $\alpha = \{\tau, \sigma, m, l\}$ diagonalize all the operators, *i.e.*,

$$\left\langle \mathbf{u}_\alpha^{(1)}, \mathcal{Z}\left[\mathbf{u}_{\alpha'}^{(1)}\right] \right\rangle = p_\alpha \delta_{\alpha\alpha'} \tag{14}$$

- Solution found by setting all waves to radiate unitary power, $\left\langle \mathbf{u}_\alpha^{(1)}, \mathcal{R}\left[\mathbf{u}_{\alpha'}^{(1)}\right] \right\rangle = 2\delta_{\alpha\alpha'}$.

Problem $\mathcal{P}_1$ ($ka \ll 1$)

- Dominant TM mode

$$\min_{\mathbf{I}} \delta_{\mathrm{loss}} = \frac{9}{4}\frac{R_\mathrm{s}}{Z_0}\frac{1}{(ka)^2}.$$

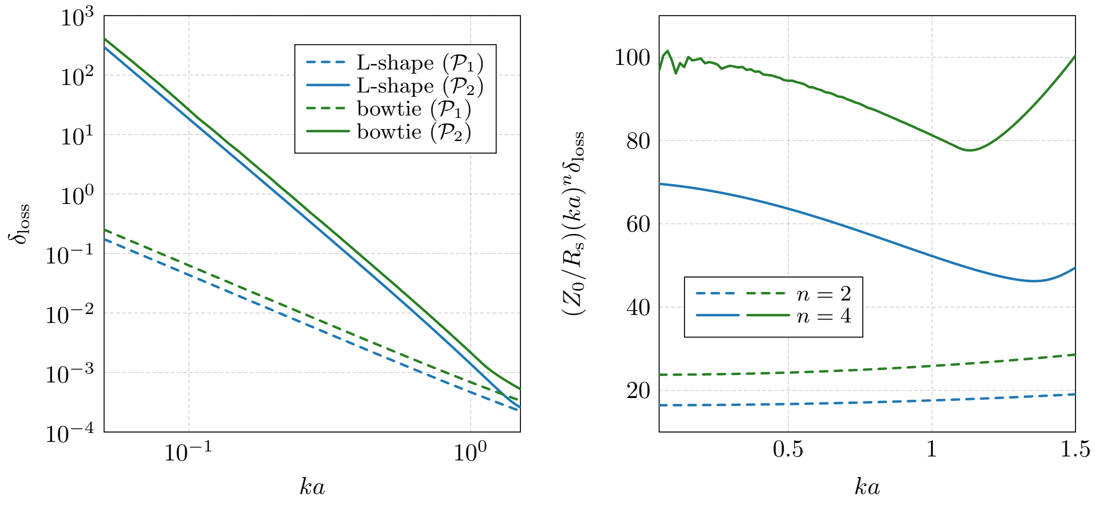
Problem $\mathcal{P}_2$ ($ka \ll 1$)

- TM and TE modes tuned to resonance

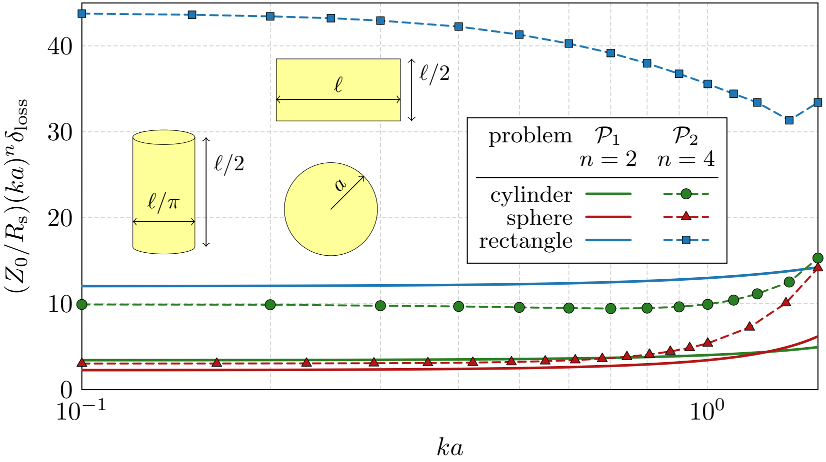
$$\min_{\mathbf{I}} \delta_{\mathrm{loss}} = 3\frac{R_\mathrm{s}}{Z_0}\frac{1}{(ka)^4}.$$

- Notice different scaling of problem $\mathcal{P}_1$ and $\mathcal{P}_2$,
- linear trade-off between normalized δ_{loss} and Q-factor.

Example: Scaling of the Problem $\mathcal{P}_1$ and $\mathcal{P}_2$



Scaling of the Problem $\mathcal{P}_1$ and $\mathcal{P}_2$



The cost of self-resonance is severe and cannot be circumvented[9].

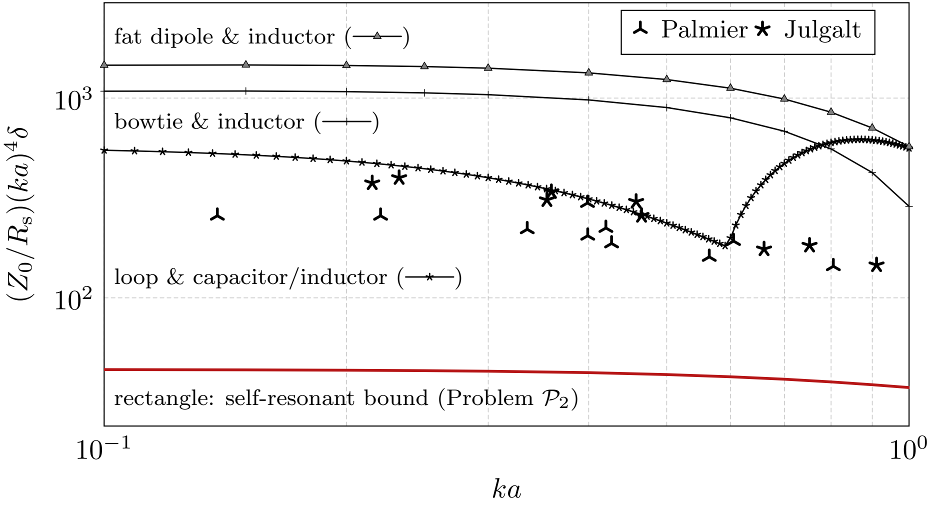
$$\mathcal{P}_1\colon\ \delta_{\mathrm{loss}} \propto \frac{R_\mathrm{s}}{Z_0}\frac{1}{(ka)^2}$$

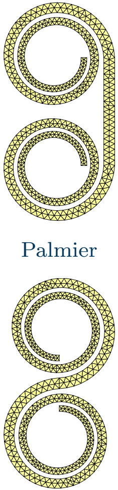
$$\mathcal{P}_2\colon\ \delta_{\mathrm{loss}} \propto \frac{R_\mathrm{s}}{Z_0}\frac{1}{(ka)^4}$$

- What about volumetric cases?

[9] L. Jelinek, K. Schab, and M. Capek, "The radiation efficiency cost of resonance tuning," *IEEE Trans. Antennas Propag.*, vol. 66, no. 12, pp. 6716 –6723, 2018

Comparison of Antennas with the Bound $\mathcal{P}_2$



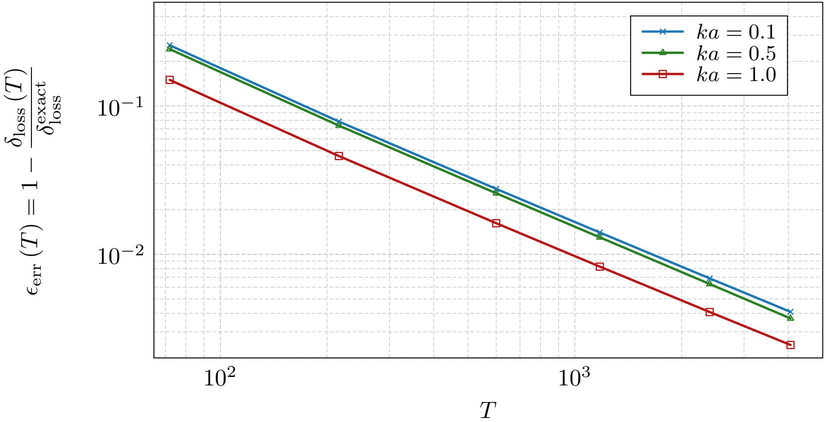


Palmier

Julgalt

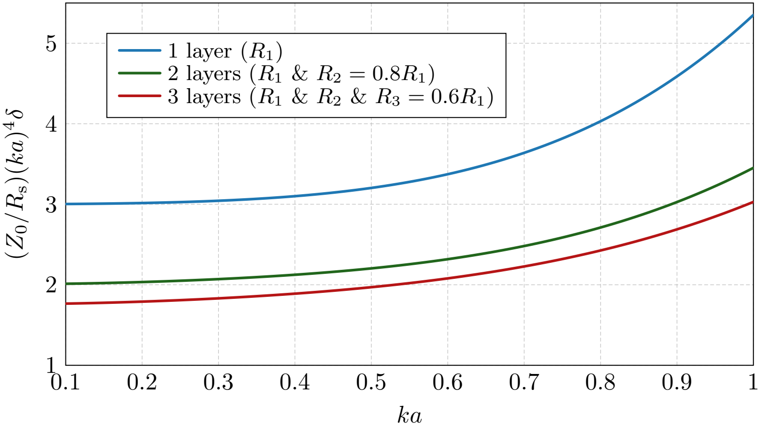
Precision of the Algebraic Formulation

- Bound corresponding to a spherical shell of radius a, compared with the analytical results.



Evaluated in AToM for $T = \{72, 216, 600, 1176, 2400, 4056\}$ triangles.

A Multi-Layered Sphere

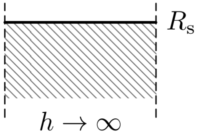


- Two spherical layers still evaluated analytically[10].
- It is confirmed that (pseudo-)volumetric current exhibits better than surface current[11].

[10] V. Losenicky, L. Jelinek, M. Capek, *et al.*, "Dissipation factors of spherical current modes on multiple spherical layers," *IEEE Trans. Antennas and Propag.*, vol. 66, no. 9, pp. 4948–4952, 2018

[11] A. Karlsson, "On the efficiency and gain of antennas," *Prog. Electromagn. Res.*, vol. 136, pp. 479–494, 2013

Limits of the Surface Resistivity Model



Ohmic losses in MoM are approximated with surface resistivity model.

- Skin depth lower than sheet's thickness ($\delta \ll h$).
- Skin depth negligible as compared to effective curvature.

Significant errors when sheets close to each other (*e.g.*, folded dipole).

- Surface resistivity model can be improved:
  - Summation of current wave and its reflection.
  - Two sheets with half resistivity (but twice as many unknowns).
  - Always problem dependent solution.

The only general remedy is a full-wave volumetric method of moments (with crazily many discretization elements for conductors).

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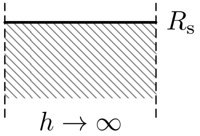

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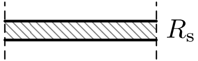

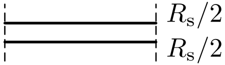

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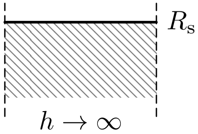

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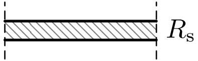

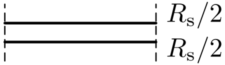

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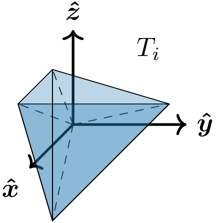
Implementation of Volumetric Method of Moments (VMoM)



VMoM implemented within periodic workshops on small antennas[12].

- Volumetric radiation integrals converted to surface integrals only[13].

$$Z_{mn} = -\mathrm{j}\frac{Z_0}{k}\int\limits_{V_{m-}} \boldsymbol{\psi}_m(\boldsymbol{r}) \cdot \left(\mathbf{1} + \boldsymbol{\chi}^{-1}(\boldsymbol{r})\right) \cdot \boldsymbol{\psi}_n(\boldsymbol{r})\,\mathrm{d}V$$

$$-\mathrm{j}\frac{Z_0}{k}\oint\limits_{S_{m-}}\oint\limits_{S_{n-}} \hat{\boldsymbol{n}}_m(\boldsymbol{r}) \cdot \Big(\boldsymbol{\psi}_m(\boldsymbol{r}) \times \left(\boldsymbol{\psi}_n(\boldsymbol{r}') \times \hat{\boldsymbol{n}}_n(\boldsymbol{r}')\right)\Big) G(\boldsymbol{r},\boldsymbol{r}')\,\mathrm{d}S'\,\mathrm{d}S$$

- Precise and fast evaluation of all (potentially) singular integrals[14].
- Constant basis functions in a center of tetrahedra $\{\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}}\} \rightarrow$ fast evaluation.

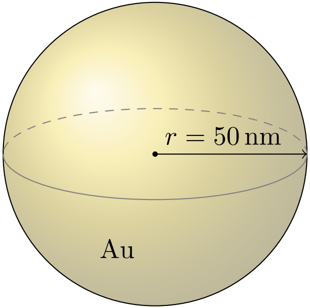
---

[12] Series of ESA Workshops.

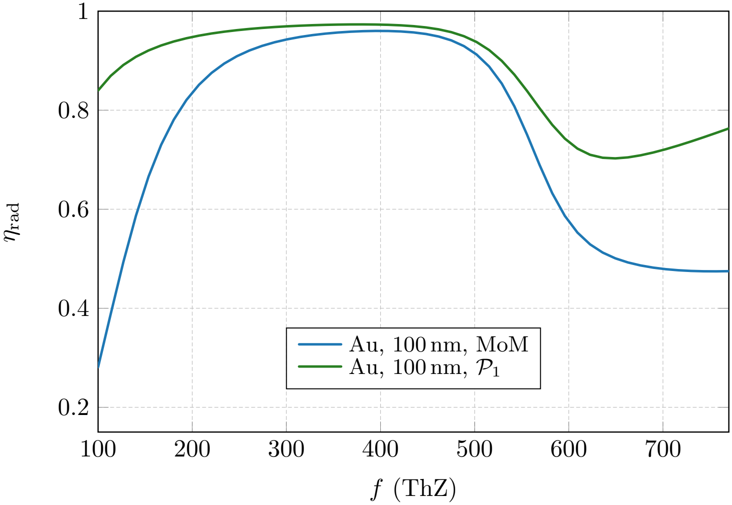
[13] A. Polimeridis, J. Villena, L. Daniel, *et al.*, “Stable FFT-JVIE solvers for fast analysis of highly inhomogeneous dielectric objects,” *Journal of Computational Physics*, vol. 269, pp. 280–296, 2014

[14] R. D. Graglia, “On the numerical integration of the linear shape functions times the 3-D green’s function of its gradient on a plane triangle,” *IEEE Trans. Antennas Propag.*, vol. 41, pp. 1448–1455, 1993

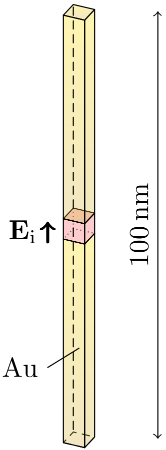
Example: Scattering of a Gold Nanoparticle (VMoM)



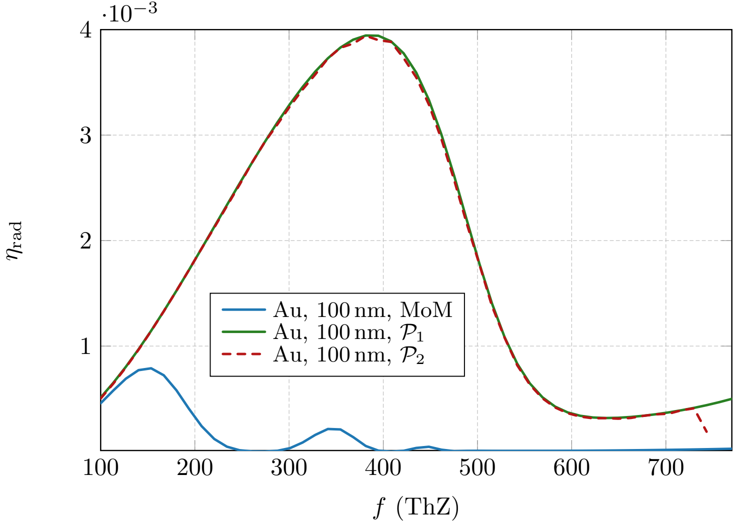
A nanoparticle excited by impinging plane wave.



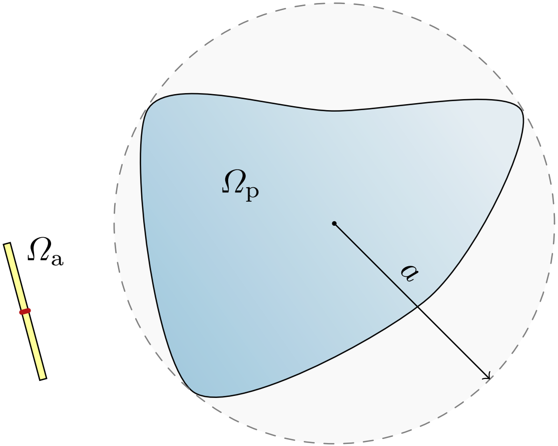
Example: Plasmonic Nanoantenna (VMoM)



A nanoantenna (a rod) fed in the middle, $L/W = 50$.



MoM & T-Matrix: Active Element Outside



Active (yellow) and passive (blue) scatterers.

- Active element modeled with MoM ($\mathbf{Z}$).
- Passive scatterer with T-matrix ($\mathbf{T}$).

$$\begin{pmatrix} \mathbf{Z} & -\mathbf{S}_4^\mathrm{T} & \mathbf{0} \\ \mathbf{S}_4 & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & -\mathbf{T} \end{pmatrix} \begin{pmatrix} \mathbf{I} \\ \mathbf{f}_1 \\ \mathbf{a}_1 \end{pmatrix} = \begin{pmatrix} \mathbf{V} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

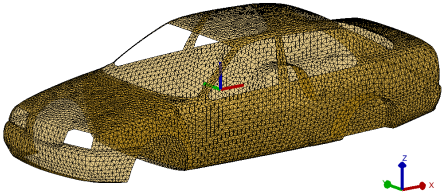
Coupling (outcoming waves):

$$S_{4,\alpha n} = k\sqrt{Z_0} \int\limits_{\Omega} \mathbf{u}_\alpha^{(4)}\left(k\boldsymbol{r}\right) \cdot \boldsymbol{\psi}_n\left(\boldsymbol{r}\right) \, \mathrm{d}S.$$

Auxiliary equation:

$$\mathbf{f}_1 = \mathbf{T}\mathbf{a}_1.$$

Example: A Dipole Antenna Close to a Car Chassis

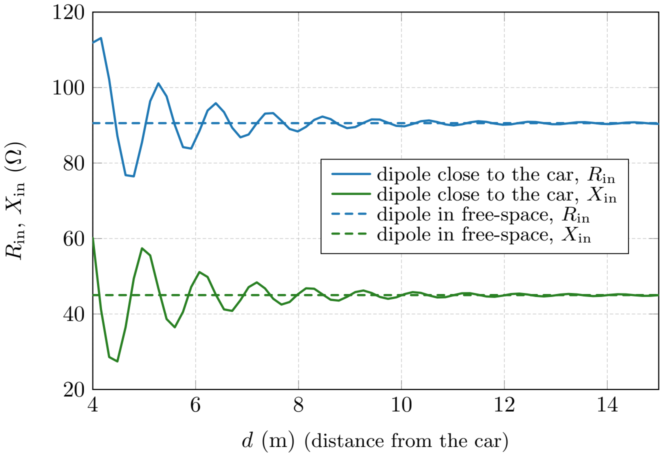


A car chassis (30426 DOF) with a half-wavelength dipole located nearby.

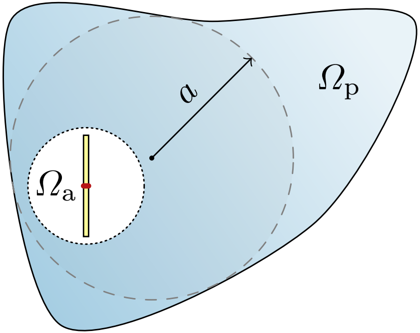
Z	**S**	**T**	total time
3980 s	299 s	252 s	4531 s

For 70 various positions of a dipole:

MoM	$70 \times 3980\,\mathrm{s} \approx$ **77.4** hours
hybrid	$4531\,\mathrm{s} + 252\,\mathrm{s} \approx$ **1.3** hours



MoM & T-Matrix: Active Element Inside



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$$\begin{pmatrix} \mathbf{Z} & -\mathbf{S}_1^\mathrm{T} & \mathbf{0} \\ \mathbf{S}_1 & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & -\mathbf{\Gamma} \end{pmatrix} \begin{pmatrix} \mathbf{I} \\ -\mathbf{a}_1 \\ -\mathbf{f}_1 \end{pmatrix} = \begin{pmatrix} \mathbf{V} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

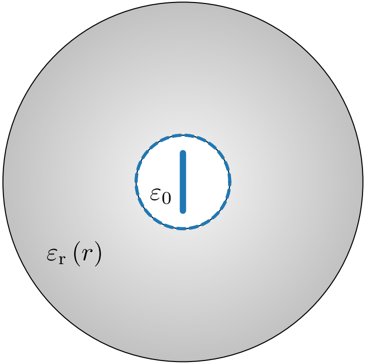
Coupling (regular waves):

$$S_{1,\alpha n} = k\sqrt{Z_0} \int\limits_{\Omega} \mathbf{u}_\alpha^{(1)} (k\boldsymbol{r}) \cdot \boldsymbol{\psi}_n (\boldsymbol{r}) \, \mathrm{d}S.$$

Auxiliary equation:

$$\mathbf{a}_1 = \mathbf{\Gamma}\mathbf{f}_1.$$

Example: Dipole in a Capsule Inside Human Body



An electrically small antenna inside capsule implanted in a body.

Results to be presented in a few days/during the conference.

MoM & T-Matrix: Comparison

- Formally similar problems to deal with (external feeding omitted here).

External case

$$\begin{pmatrix} \mathbf{Z} & -\mathbf{S}_4^{\mathrm{T}} & \mathbf{0} \\ \mathbf{S}_4 & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & -\mathbf{T} \end{pmatrix} \begin{pmatrix} \mathbf{I} \\ \mathbf{f}_1 \\ \mathbf{a}_1 \end{pmatrix} = \begin{pmatrix} \mathbf{V} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

- Creeping waves,
- devices close to human body,
- small devices close to large platforms.

Internal case

$$\begin{pmatrix} \mathbf{Z} & -\mathbf{S}_1^{\mathrm{T}} & \mathbf{0} \\ \mathbf{S}_1 & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & -\boldsymbol{\Gamma} \end{pmatrix} \begin{pmatrix} \mathbf{I} \\ -\mathbf{a}_1 \\ -\mathbf{f}_1 \end{pmatrix} = \begin{pmatrix} \mathbf{V} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

- Implantable antennas,
- special lenses.

Concluding Remarks

- Integral equations and MoM is about more than just $\mathbf{I} = \mathbf{Z}^{-1}\mathbf{V}$!
- MoM-related operators $(\mathbf{Z}, \mathbf{W}, \mathbf{S}, \mathbf{U}, \mathbf{L}, \dots)$ have unthought applications.

What has been done

- Bounds on radiation efficiency well understood.
- Cost of self-resonance evaluated.
- Trade-offs with Q-factor and antenna gain known.

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Topics of ongoing research

- Improved model for surface resistivity.
- Finalization of MoM–T-matrix hybrid method.
- Tightness of the bounds (topo. sensitivity check, number of ports).
- SMoM+VMoM (good conductors immersed in material).

Questions?

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ICECOM, Dubrovnik, Croatia
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The presentation is downloadable at ▶ capek.elmag.org

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